ECS 455: Quiz 4

Instructions

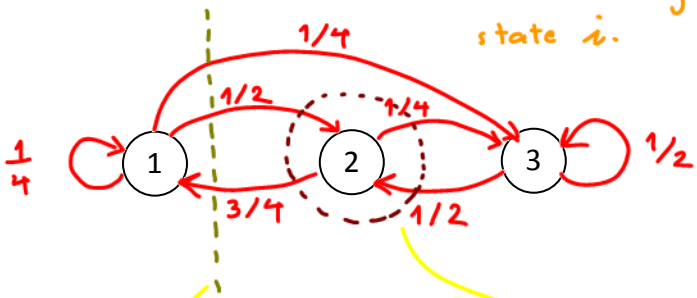
1. Separate into groups of no more than three persons.
2. Only one submission is needed for each group. Late submission will not be accepted.
3. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
4. **Do not panic.**

Name	ID
Prapun	

Consider a Markov chain whose transition probability matrix is given by $\mathbf{P} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1/4 & 1/2 & 1/4 \\ 3/4 & 0 & 1/4 \\ 0 & 1/2 & 1/2 \end{bmatrix} \end{array}$.

The (i,j) element in P gives the probability of the system evolving to state j in the next "slot" given that it is currently in state i.

1. Draw the corresponding Markov chain



2. In your drawing above, draw (using a different color) the boundary that corresponds to the balance equation $\frac{1}{2}p_1 - p_2 + \frac{1}{2}p_3 = 0$. $\Leftrightarrow$ $\frac{1}{2}p_1 + \frac{1}{2}p_3 = p_2$ $\Rightarrow$ The corresponding boundary is which is simply the one enclosing state (2).

   state (1) and state (3) on one side; state (2) on another side

3. Find the steady-state probabilities for this Markov chain. (Don't forget to indicate the boundaries and the corresponding balance equations.)

There are three unknowns: p_1, p_2, p_3. So, we need three equations.
We always have one: $p_1 + p_2 + p_3 = 1$. $\Rightarrow$ $p_1 + p_3 = 1 - p_2$

Previously, we already have one boundary: $\frac{1}{2}p_1 + \frac{1}{2}p_3 = p_2$

$1 - p_2 = 2p_2$

$p_2 = 1/3$

We draw another boundary in the Markov chain to get $\frac{1}{4}p_1 + \frac{1}{2}p_1 = \frac{3}{4}p_2$

$p_1 = p_2 = \frac{1}{3}$

$p_3 = 1 - p_1 - p_2 = \frac{1}{3}$.

Therefore, $p_1 = p_2 = p_3 = 1/3$